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Eisenstein type integrals on sphere and their generalizations

Eigenfunctions of the distorted Laplacian on the two-dimensional sphere are studied. Integral representations for homogeneous eigenfunctions of this operator with a Poisson type kernel are obtained.

For $k \in \mathbb{Z}$ we set

$$\mathfrak{D}_k = \begin{cases} \frac{1}{k!}\left(\frac{\partial}{\partial z}\right)^k, & k \geq 0 \\ \frac{1}{(-k)!}\left(\frac{\partial}{\partial \overline{z}}\right)^{-k}, & k < 0. \end{cases}$$

Let $\lambda \in \mathbb{C}$, $\nu = \nu(\lambda) = \frac{1-\lambda}{2}$, $\eta \in \mathbb{S}^1$. We define a function $E^s_{\nu,\,\eta}$ by the equality

$$E^s_{\nu,\,\eta}(z) = \left(\frac{1+|z|^2}{1-i\eta\overline{z}-i\overline{\eta}z-|z|^2}\right)^{\nu}\left(\frac{1-i\eta\overline{z}}{1-i\overline{\eta}z}\right)^s, \quad |z|<1.$$

Denote by $H^s_{\lambda,\,k}(t)$ the function

$$t^{|k|}(1+t^2)^{\nu}F\left(\nu-s+\frac{|k|-k}{2},\, \nu+s+\frac{|k|+k}{2};\, |k|+1;\, -t^2\right),$$

where F is the Gauss hypergeometrical function.

Theorem 1. *Let*

$$\mathfrak{L} = 4(1+|z|^2)^2\frac{\partial^2}{\partial z\partial\overline{z}} - 4s^2|z|^2 Id - 4s(1+|z|^2)\left(z\frac{\partial}{\partial z}-\overline{z}\frac{\partial}{\partial\overline{z}}\right),$$

where Id is the identity mapping.

(i) *Any smooth function $f : \mathbb{C} \to \mathbb{C}$ satisfying the equation*

$$\mathfrak{L}f = (4s^2+1-\lambda^2)f$$

and the condition

$$f(e^{i\theta}z) = e^{ik\theta}f(z)$$

has the form $f(z) = (\mathfrak{D}_k f)(0)H^s_{\lambda,\,k}(|z|)\left(\frac{z}{|z|}\right)^k$.

(ii) *The following equality*

$$\frac{1}{2\pi}\int_0^{2\pi} E^s_{\nu,\,e^{i\theta}}(z)e^{ik\theta}d\theta = C_{\nu,\,k,\,s}H^s_{\lambda,\,k}(|z|)\left(\frac{z}{|z|}\right)^k, \quad |z|<1,$$

holds, where

$$C_{\nu,\,k,\,s} = \begin{cases} \frac{\Gamma(\nu+s+k)}{k!\Gamma(\nu+s)}, & k \geq 0; \\ \frac{\Gamma(\nu-s-k)}{(-k!)\Gamma(\nu-s)}, & k < 0. \end{cases}$$

The representations of this form called Eisenstein's integrals play an important role in various questions of the analysis and applications.

For similar results for eigenfunctions of the Laplace-Beltrami operator on symmetric spaces you see, for example, in [1-3].

References

[1] *Volchkov V. V.* Integral Geometry and Convolution Equations. Dordrecht: Kluwer, 2003. – 454p.

[2] *Volchkov V. V., Volchkov Vit. V.* Harmonic Analysis of Mean Periodic Functions on Symmetric Spaces and the Heisenberg Group. – London :Springer, 2009. – 671p.

[3] *Volchkov V. V., Volchkov Vit. V.* Offbeat Integral Geometry on Symmetric Spases. – Basel: Birkhåuser, 2013. – 592p.